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LETTER TO THE EDITOR

Symmetries of the stochastic Burgers equation

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Abstract. All Lie symmetries of the Burgers equation driven by an external random force are found. Besides the generalized Galilean transformations, this equation is also invariant under the time reparametrizations. It is shown that the Gaussian distribution of a pumping force is not invariant under the symmetries and breaks them down leading to the non-trivial vacuum (instanton). Integration over the volume of the symmetry groups provides the description of fluctuations around the instanton and leads to an exactly solvable quantum mechanical problem.

1. Introduction

The statistical theory of turbulence was put forward by Kolmogorov in 1941 [1] and has since been developed intensively. The cornerstone of although simple but suprisingly robust Kolmogorov's dimensional analysis is the assumption that in the fully developed turbulence there is a range of scales where the velocity structure functions are universal, i.e. independent of the cutoffs provided by the scales of energy pumping and dissipation. Much effort has been made to understand whether there are fluctuation corrections to the mean field scaling exponents, predicted by Kolmogorov, and whether these corrections depend on the dissipation or pumping scale [2]. Nevertheless, the problem is still far from being solved.

Recently, the one-dimensional Burgers equation

$$u_t + uu_x - \nu u_{xx} = f(x,t) \tag{1}$$

driven by the Gaussian random force $f(x,t)$ with the zero mean and covariance

$$\langle f(x,t) f(x',t')\rangle = \kappa(x-x')\delta(t-t') \tag{2}$$

has been in the focus of quite a number of studies. The reason is that this equation is the simplest one that resembles the analytic structure of the Navier–Stokes equation and, at least formally, is within the scope of applicability of the Kolmogorov's arguments. Extensive numerical simulations of this equation [3], although reproducing Kolmogorov's scaling exponents for the energy spectrum, and for the two-point velocity correlation function, reveal strong intermittency for high-order moments of velocity differences.

By adopting the hypothesis of the existence of the operator product expansion in the limit of small but non-zero viscosity, $\nu \to +0$, Polyakov [4] reduced the problem of calculation of high-order moments to an exactly solvable quantum mechanics and qualitatively explained the results of numerical simulations.

Another approach was based on the equivalence of the stochastic Burgers equation, equations (1) and (2), with the Martin–Siggia–Rose field theory. Employing the saddle-point approximation in the corresponding path integral, equation (14), Gurarie and Migdal [5]

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found the same generating functional of the velocity correlation functions as was predicted by Polyakov. The classical solution that provides a minimum of the action was named the 'instanton'.

The purpose of this letter is to show that the Burgers equation driven by an external random force (1), possesses two infinite symmetry groups that are broken by the Gaussian distribution of a pumping force (2), thus resulting in the instanton solution. The exact integration over the volumes of the symmetry group corresponds to the description of fluctuations around the instanton and leads finally to the Polyakov's exactly solvable quantum mechanics.

1.1. Martin–Siggia–Rose formalism

Let us first consider the functional Langevin equation of the general type

$$\partial_t u + \mathcal{F}(x, [u]) = f(x, t) \tag{3}$$

where $f(x, t)$ is a functional Gaussian process with the covariance operator K. The local functional $\mathcal{F}(x, [u])$ completely characterizes the dynamics of the system.

Then, the Martin–Siggia–Rose formalism permits one to obtain a formal path integral representation for the expectation values of the observables $\mathcal{O}[u]$ in the stationary state [6–10]

$$\begin{aligned}\langle \mathcal{O}[u] \rangle &= \int \mathcal{D}f\, \mathcal{O}[u] \delta(\partial_t u + \mathcal{F} - f) \mathrm{e}^{-(f, K^{-1} f)/2} \\ &= \int \mathcal{D}\mu\, \mathcal{D}f\, \mathcal{O}[u] \delta(\partial_t u + \mathcal{F} - f) \mathrm{e}^{-(\mu, K\mu)/2 + \mathrm{i}(\mu, f)}\end{aligned} \tag{4}$$

where we have introduced an auxiliary field μ to rewrite the Gaussian path integral over the field f. Now, let us pass in this path integral from the integration over the force field $\mathcal{D}f$ to the integration over the velocity field $\mathcal{D}u$. Using the δ-function we obtain

$$\langle \mathcal{O}[u] \rangle = \int \mathcal{D}\mu\, \mathcal{D}u\, \mathcal{O}[u] J[u] \exp(-S[u, \mu]). \tag{5}$$

Now, the action of the effective field theory can be written as

$$S[u, \mu] = -\mathrm{i}(\mu, \partial_t u + \mathcal{F}) + \tfrac{1}{2}(\mu, K\mu). \tag{6}$$

The functional $J[u]$ is the Jacobian of the force-to-velocity transformation and is given by the following expression:

$$J[u] = \det \left\| \frac{\delta f}{\delta u} \right\| = \det \left\| \partial_t + \frac{\delta \mathcal{F}}{\delta u} \right\|. \tag{7}$$

Here, the variational derivative $\delta\mathcal{F}/\delta u$ is determined by the usual equality

$$\mathcal{F}(x, [u + \delta u]) - \mathcal{F}(x, [u]) = \int \frac{\delta \mathcal{F}(x, [u])}{\delta u(y)} \delta u(y)\, \mathrm{d}y. \tag{8}$$

Formal calculation of this determinant leads to the result

$$J[u] = \exp\left(\theta(0) \mathrm{Tr} \frac{\delta \mathcal{F}}{\delta u}\right) \tag{9}$$

involving the ill defined quantity $\theta(0)$, where $\theta(t)$ is the usual step function. To give a meaning to this expression one can make use of the usual limiting procedure in which the time interval (t_i, t_f) is subdivided into N equal subintervals each of duration Δt, and the differential relation inside is replaced by a difference one [10]. After taking the continuous limit one finds $\theta(0) = 1/2$.

In the perturbative treatment of the path integral this Jacobian term is irrelevant because it can always be absorbed by the proper redefinition of Green functions. Nevertheless, performing manipulations with the path integral we should take care about this Jacobian term. For the stochastic Burgers equation the local functional $\mathcal{F}(x,[u])$ can be defined as follows:

$$\mathcal{F}(x,[u]) = \int \mathrm{d}y\, \delta(x-y)(uu_y - \nu u_{yy}). \tag{10}$$

Its variational derivative is then equal to

$$\frac{\delta\mathcal{F}(x,[u])}{\delta u(y)} = \delta'(x-y)u - \nu\delta''(x-y) \tag{11}$$

and the trace term formally becomes

$$\mathrm{Tr}\frac{\delta\mathcal{F}}{\delta u} = \int \mathrm{d}t\,\mathrm{d}x\, \frac{\delta\mathcal{F}(x,[u])}{\delta u(x)} = \delta'(0)\int \mathrm{d}t\,\mathrm{d}x\, u - \nu\delta''(0)\int \mathrm{d}t\,\mathrm{d}x. \tag{12}$$

Here, the first term vanishes for symmetry reasons and the second term does not depend on u and can be absorbed by the normalization factor in the path integral. Hence, for the Burgers equation the Jacobian of the force-to-velocity transformation is equal to some irrelevant constant.

So, the generating functional of the velocity correlation functions

$$\langle F[\lambda]\rangle = \left\langle \exp\int \mathrm{d}x\, \lambda(x)u(x,0)\right\rangle \qquad \int \mathrm{d}x\, \lambda(x) = 0 \tag{13}$$

can be represented by the path integral

$$\langle F[\lambda]\rangle = \int \mathcal{D}\mu\mathcal{D}u\, F[\lambda]\exp(-S[u,\mu]) \tag{14}$$

where the action of the effective field theory is equal to

$$S[u,\mu] = -\mathrm{i}\int \mathrm{d}t\,\mathrm{d}x\, \mu(u_t + uu_x - \nu u_{xx}) + \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}x'\mu(x,t)\kappa(x-x')\mu(x',t) \tag{15}$$

and time integration runs over the interval (t_i, t_f). For the sake of simplicity, we will consider the time interval $(-\infty, 0)$ although the following does not depend on the particular choice. $\kappa(x)$ is supposed to be a slowly varying even function with the expansion

$$\kappa(x) = \kappa(0) - \frac{\kappa_0}{2}x^2 \qquad |x| \ll \sqrt{\frac{\kappa(0)}{\kappa_0}} \equiv l \tag{16}$$

and quickly turns to zero when $|x| \gg l$. The interval l characterizes the correlation length of the random force and we only study velocity correlation functions within this interval.

2. Symmetries of the Burgers equation

We start our consideration with a quite natural question: what is the most general transformation leaving invariant the Burgers equation driven by an external force, equation (1), or, in other words, given two functions $u(x,t)$ and $f(x,t)$ satisfying the Burgers equation, what is the most general transformation that produces another pair of functions satisfying the same equation?

Making use of the methods of Lie group analysis of differential equations [11] it is possible to prove that all such transformations form the symmetry group which consists of the two infinite subgroups, generalized Galilean transformations (G),

$$\check{t} = t \tag{17a}$$
$$\check{x} = x + a(t) \tag{17b}$$
$$\check{u} = u + a'(t) \tag{17c}$$
$$\check{f} = f + a''(t) \tag{17d}$$

and time reparametrizations (L),

$$\tilde{t} = b(t) \tag{18a}$$
$$\tilde{x} = x\sqrt{b'(t)} \tag{18b}$$
$$\tilde{u} = \left(u + \frac{x}{2}\frac{b''(t)}{b'(t)}\right)\Bigg/\sqrt{b'(t)} \tag{18c}$$
$$\tilde{f} = \left(f + \frac{x}{2}\{b(t), t\}\right)\Bigg/\sqrt{b'(t)^3} \tag{18d}$$

where $a(t)$ is an arbitrary function, $b(t)$ maps the time interval (t_i, t_f) onto itself and $b'(t) > 0$. The braces

$$\{b(t), t\} = \frac{b'''}{b'} - \frac{3}{2}\left(\frac{b''}{b'}\right)^2 \tag{19}$$

are known in the theory of complex functions as the 'Schwarzian derivative'.

Symmetry transformations (17) and (18), although not depending explicitly on the viscosity ν, are stipulated by the structure of the diffusion term. In particular, these transformations leave also invariant, along with the Burgers equation itself, the following relation,

$$(\mathrm{d}x - u\mathrm{d}t)^2 \sim \mathrm{d}t \tag{20}$$

which coincides (up to convective term, $u\mathrm{d}t$) with a similar relation for a diffusion process.

The symmetry groups corresponding to the transformations (17) and (18) are generated by the infinitesimal operators

$$G(\alpha(t)) = \alpha\frac{\partial}{\partial x} + \alpha'\frac{\partial}{\partial u} + \alpha''\frac{\partial}{\partial f} \tag{21a}$$
$$L(\beta(t)) = \beta\frac{\partial}{\partial t} + \frac{1}{2}\beta' x\frac{\partial}{\partial x} + \frac{1}{2}(\beta'' x - \beta' u)\frac{\partial}{\partial u} + \frac{1}{2}(\beta''' x - 3\beta'' f)\frac{\partial}{\partial f} \tag{21b}$$

which, in the basis

$$G_r = G(t^{r+1/2}) \qquad L_n = L(t^{n+1}) \tag{22}$$

form a Lie algebra with the commutation relations

$$[G_r, G_s] = 0 \tag{23a}$$
$$[L_n, G_s] = (s - n/2)G_{n+s} \tag{23b}$$
$$[L_n, L_m] = (m - n)L_{n+m} \tag{23c}$$

where n, m are integers and r, s are half-integers.

3. Action transformation laws

Up until now, we have not been concerned about the probability distribution of the pumping force. However, if the force $f(x,t)$ is a Gaussian random function, then its probability measure is not invariant under the symmetry transformations. It is easy to check directly that the action (15) which in fact defines such a probability measure changes under the generalized Galilean transformations as

$$\check{S}[\check{u},\check{\mu}] = S[u,\mu] - \mathrm{i}\int \mathrm{d}t\,\mathrm{d}x\,\mu(x,t)a''(t) \tag{24}$$

and under the time reparametrizations as

$$\tilde{S}[\tilde{u},\tilde{\mu}] = -\mathrm{i}\int \mathrm{d}t\,\mathrm{d}x\,\mu(u_t + uu_x - \nu u_{xx}) + \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}x'\,b'^2\mu(x,t)\kappa(\sqrt{b'}(x-x'))\mu(x',t)$$
$$-\frac{\mathrm{i}}{2}\int \mathrm{d}t\,\mathrm{d}x\,x\mu(x,t)\{b(t),t\}. \tag{25}$$

Hence, we come to the conclusion that it is the Gaussian distribution of the pumping force which breaks down the infinite symmetry group of the Burgers equation (1).

The action is still invariant under the finite subgroup which consists of spatial translations generated by the infinitesimal operator $G_{-1/2}$, Galilean transformations generated by the operator $G_{1/2}$, and time translations with the generator L_{-1}. According to the Noether theorem [11] there are three conservation laws corresponding to the three-parameter subgroup of variational symmetries, the momentum conservation

$$P = -\mathrm{i}\int \mathrm{d}x\,\mu u_x \tag{26}$$

the conservation of the centre-of-mass motion

$$M = \mathrm{i}\int \mathrm{d}x\,\mu + tP \tag{27}$$

and the energy conservation

$$E = \mathrm{i}\int \mathrm{d}x\,\mu u_t + L \tag{28}$$

where L is the Lagrangian $S = \int \mathrm{d}t\,L$.

4. The Faddeev–Popov method

If the action were invariant with respect to the infinite symmetry group, we could employ the Faddeev–Popov method to eliminate the corresponding degrees of freedom [10]. As we will see later, in our case the same method separates the degrees of freedom of the symmetry group transformations and leads finally to the exactly solvable quantum mechanics for the separated modes. The Faddeev–Popov method consists of three steps. At first, considering the field theory

$$Z = \int \mathcal{D}A\,\exp(-S[A]) \tag{29}$$

with the action, $S[A]$, invariant under the group $\mathcal{G}$ of gauge transformations $S[A^g] = S[A]$, we define the equation $f(A) = 0$ which fixes uniquely the gauge degrees of freedom, i.e. the equation $f(A^g) = 0$ should provide the unique solution for the group element g for an arbitrary chosen field configuration A. Then, we define the functional Jacobian $J_{\mathcal{G}}[A]$ by the condition

$$J_{\mathcal{G}}[A] \int \mathcal{D}g\, \delta(f(A^g)) = 1 \tag{30}$$

where $J_{\mathcal{G}}[A] = \det \|\delta f(A^g)/\delta g\|$, and integration runs over the volume of the group $\mathcal{G}$. It is clear that the functional $J_{\mathcal{G}}[A]$ is invariant by construction under the gauge transformations, $J_{\mathcal{G}}[A^g] = J_{\mathcal{G}}[A]$. So, we can calculate the Jacobian only for the identity element of the group. Finally, if we insert the identity (30) into the functional integral (29) and shift the variables $A^g \to A$, we separate the degrees of freedom corresponding to the group volume

$$Z = \int \mathcal{D}A\, J_{\mathcal{G}}[A]\delta(f(A)) \exp(-S[A]) \int \mathcal{D}g. \tag{31}$$

Now, we can apply the same method to integrate over the volume of the groups of generalized Galilean transformations and time reparametrizations.

5. Integration over the volume of the group G

The gauge fixing equation in this case is $u(0, t) = 0$, and we consider the identity

$$J_G[u] \int \mathcal{D}a\,(t)\delta(a' - u(a, t)) = 1 \tag{32}$$

which is the definition of the Jacobian $J_G[u]$, integration runs over the volume of the group G. The explicit form of the Jacobian, J_G, is given by the functional determinant

$$J_G[u] = \det \left\| \partial_t - \frac{\delta u(a(t), t)}{\delta a(t)} \right\|. \tag{33}$$

For the identity element of the group, $a(t) = 0$, this expression takes the form

$$J_G[u] = \det \|\partial_t - u_x(0, t)\|. \tag{34}$$

To preserve the causality of the Langevin equation after the generalized Galilean transformation we should propagate the operator, whose determinant we are calculating, backward in time. Standard calculation [10] then gives the result

$$J_G[u] = \exp\left((1 - \theta(0)) \int \mathrm{d}t\, u_x(0, t)\right) \tag{35}$$

involving the same ill defined quantity $\theta(0)$ that appeared in the derivation of the Martin–Siggia–Rose field theory, equation (9). As we will see later, the choice $\theta(0) = 1/2$ is self-consistent in the sense that it ensures finally the existence of the steady state. Actually, the value of $\theta(0)$ plays exactly the same role as the *B-anomaly* term in the Polyakov operator product expansion.

Now, we insert the identity (32) into the functional integral (13). Then, after the properly defined generalized Galilean transformation (17), we turn the argument of the δ-function into the gauge fixing term, i.e. turn it into the equation of the surface in the functional space that intersects the orbits of the symmetry group only once. Finally, we get for the generating functional of the velocity correlation functions (13)

$$\langle F[\lambda] \rangle = \int \mathcal{D}\mu \int \mathcal{D}a\ \exp(-S_G[a, \mu]) \int \mathcal{D}u\, \delta(u(0)) J_G F[\lambda] \exp(-S[u, \mu]) \tag{36}$$

where we have used the condition $\int \mathrm{d}x\, \lambda(x) = 0$, which ensures the invariance of the functional $F[\lambda]$ under the generalized Galilean transformations,

$$S_G[a, \mu] = -\mathrm{i} \int \mathrm{d}t\, \mathrm{d}x\, \mu(x, t) a''(t) \tag{37}$$

is the effective action of the corresponding modes. In fact, all these modes can be integrated out, leading to the additional constraint

$$\int \mathcal{D}a(t) \exp(-S_G[a, \mu]) = \delta\left(\partial_t^2 \int \mathrm{d}x\, \mu\right). \tag{38}$$

From this it follows that the zero moment of the auxiliary field μ is just a linear function of time

$$\pi_0 = \int \mathrm{d}x\, \mu = \mathrm{i}(Pt - M) \tag{39}$$

where P and M are the integrals of motion (26) and (27). In the frame of reference moving together with the centre-of-mass of the Burgers fluid both of the integrals are equal to zero and $\pi_0 \equiv 0$.

The integration over the volume of the group of generalized Galilean transformations has first been proposed [5, 12] to justify the stability of the saddle-point approximation in the path integral.

6. Integration over the volume of the group L

The gauge fixing equation is $u(x_0, t) = 0$, where $x_0 \neq 0$, and we consider the identity

$$J_L[u] \int \mathcal{D}b(t)\, \delta\left(\frac{x_0}{2}\frac{b''}{b'} - u(x_0\sqrt{b'}, b)\sqrt{b'}\right) = 1. \tag{40}$$

The explicit expression for the Jacobian J_L is

$$J_L[u] = \det\left\| \frac{\delta}{\delta b(t)} \left(\frac{x_0}{2}\frac{b''}{b'} - u(x_0\sqrt{b'}, b)\sqrt{b'}\right)\right\| \tag{41}$$

which in the new variables

$$\ln\sqrt{b'(t)} = \sigma(b) \qquad \frac{b''(t)}{2b'(t)^2} = \sigma'(b) \tag{42}$$

can be rewritten as

$$J_L[u] = \det\left\| \frac{\delta\{x_0\sigma' - u(x_0\, \mathrm{e}^{\sigma}, b)\mathrm{e}^{-\sigma}\}\, \mathrm{e}^{2\sigma}}{\delta\sigma}\right\| \det\left\|\frac{\delta\sigma}{\delta b}\right\|. \tag{43}$$

Again, due to invariance of the Jacobian under the group of time reparametrizations it can be computed only for the identity element of the group

$$J_L[u] = \det\|\partial_t - u_x(x_0, t)\| \tag{44}$$

where the unessential constant coefficient that does not depend on u is omitted and the constraint $u(x_0, t) = 0$ is used. Calculating the determinant along the lines indicated above, we get

$$J_L[u] = \exp\left(\frac{1}{2}\int \mathrm{d}t\, u_x(x_0, t)\right). \tag{45}$$

Let us now insert the identity (40) into the functional integral (36). Then, after a properly defined time reparametrization, the generating functional of the velocity correlation functions is converted into

$$\langle F[\lambda]\rangle = \int \frac{\mathcal{D}\mu}{\mathcal{D}\pi_0}\mathcal{D}u\,\delta(u(0))\delta(u(x_0))J_G J_L F[\lambda]\exp(-S_0)$$
$$\times\int \mathcal{D}b\exp\left\{-S_L[b,\pi]+\frac{b''(0)}{2b'(0)^2}\int \mathrm{d}x\,x\lambda(x)\right\} \tag{46}$$

where $\pi(t)=\int \mathrm{d}x\,x\mu(x,t)$ and

$$S_0[u,\mu] = -\mathrm{i}\int \mathrm{d}t\,\mathrm{d}x\,\mu(u_t+uu_x-\nu u_{xx}) \tag{47}$$

is the action of the modes remaining. Due to the special choice of the noise covariance equation (16), it does not depend on the pumping force.

In the limit of the small but non-zero viscosity, $\nu\to+0$, we can also separate the integration over π. The desired effective action of the separated modes $b(t)$ and $\pi(t)$,

$$S_L[b,\pi] = -\frac{\mathrm{i}}{2}\int \mathrm{d}t\,\pi(t)\{b(t),t\} - \frac{1}{2}\int \mathrm{d}t\,\frac{b''(t)}{2b'(t)} + \frac{\kappa_0}{2}\int \mathrm{d}t\,b'(t)^3\pi(t)^2 \tag{48}$$

then follows from the action transformation law under the time reparametrizations (25), and the analogous transformation law for the Jacobian

$$\tilde{J}_G[\tilde{u}] = J_G[u]\exp\left(\frac{1}{2}\int \mathrm{d}t\,\frac{b''}{2b'}\right). \tag{49}$$

Separating the modes, we obtain another ambiguous parameter. Namely, we can choose an arbitrary normalization factor, Z_0, for the partition function of the modes of the symmetry group transformations and the inverse, $1/Z_0$, for the partition function of the modes remaining. This parameter plays exactly the same role as the *A-anomaly* term in Polyakov theory. Again, the choice $Z_0=1$ is self-consistent and ensures the existence of the steady state (see also [13]).

It should be stressed at this point that the gauge fixing term $\delta(u(x_0))$ works properly only if the degrees of freedom associated with the field $a(t)$ have already been integrated out, i.e. we cannot change the order and integrate first over the volume of the group L.

7. Equivalent quantum mechanics

The action (48) is equivalent to the Polyakov exactly solvable quantum mechanics. To make it obvious, let us pass from the variables t, $b(t)$, and $\pi(t)$ to the new ones

$$b=b(t) \qquad q(b)=\frac{b''(t)}{2b'(t)^2} \qquad p(b)=b'(t)\pi(t) \tag{50}$$

and finally obtain

$$S_L[q,p] = \int \mathrm{d}b\,\left\{-\mathrm{i}p(q'+q^2) - \frac{3}{2}q + \frac{\kappa_0}{2}p^2\right\} \tag{51}$$

where the additional term, $-q$, comes from the Jacobian $\mathcal{D}\pi\,\mathcal{D}b/\mathcal{D}p\,\mathcal{D}q$. This action corresponds to the quantum mechanics (in imaginary time) with the Hamiltonian [10]

$$\boldsymbol{H} = \frac{\kappa_0}{2}\left(\boldsymbol{p}-\frac{\mathrm{i}q^2}{\kappa_0}\right)^2 + \frac{q^4}{2\kappa_0} - \frac{3q}{2}. \tag{52}$$

Following Polyakov [4], we can find the zero energy eigenfunction of this Hamiltonian and finally calculate the generating functional of the velocity correlation functions,

$$\langle F[\lambda]\rangle = \exp\left\{\frac{\sqrt{2\kappa_0}}{3}\left[\int \mathrm{d}x\, x\lambda(x)\right]^{3/2}\right\}. \tag{53}$$

In conclusion we would like to mention that the method proposed in this letter seems to have a wider applicability than just to the Burgers equation considered. Any stochastic equation of Langevin type that possesses an infinite symmetry group can be treated in this way.

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References

[1] Kolmogorov A N 1941 *C. R. Acad. Sci. USSR* **30** 301
[2] Monin A S and Yaglom A M 1975 *Statistical Fluid Mechanics* (Cambrige, MA: MIT)
[3] Chekhlov A and Yakhot V 1995 *Phys. Rev.* E **51** R2739
[4] Polyakov A M 1995 *Phys. Rev.* E **52** 6183
[5] Gurarie V and Migdal A 1996 *Phys. Rev.* E **54** 4908
Gurarie V *Report* No hep-th/9606089
[6] Martin P C, Siggia E D and Rose H A 1973 *Phys. Rev.* A **8** 423
[7] H K Janssen 1976 *Z. Phys.* B **23** 377
Phythian R 1977 *J. Phys. A: Math. Gen.* **10** 777
[8] de Dominicis C 1978 *J. Physique* **37** 247
de Dominicis C and Peliti L 1978 *Phys. Rev.* B **18** 353
[9] Gawedzki K and Kupiainen A 1995 *Lecture Notes of Schladming School of Theoretical Physics (Austria, March 4–11, 1995)* Report chao-dyn/9504002
[10] Zinn-Justin J 1993 *Field Theory and Critical Phenomena* (Oxford: Clarendon) ch 4
[11] Ibragimov N H 1985 *Transformations Group Applied to Mathematical Physics* (Dordrecht: Reidel)
[12] Balkovsky E, Falkovich G, Kolokolov I and Lebedev V 1997 *Preprint* chao-dyn/9603015 *Phys. Rev. Lett.* to be published
[13] Boldyrev S 1997 *Preprint* hep-th/9610080 *Phys. Rev.* E to be published